GOAL
Identify glide reflections in a plane and represent transformations as compositions of simpler transformations.

VOCABULARY
A glide reflection is a transformation in which every point $P$ is mapped onto a point $P''$ by the following steps:
1. A translation maps $P$ onto $P'$.
2. A reflection in a line $k$ parallel to the direction of the translation maps $P'$ onto $P''$.

When two or more transformations are combined to produce a single transformation, the result is called a composition of the transformations.

Theorem 7.6 Composition Theorem
The composition of two (or more) isometries is an isometry.

EXAMPLE 1
Finding the Image of a Glide Reflection

Use the information below to sketch the image of $\triangle ABC$ after a glide reflection.

$A(-5, 5), B(-3, 2), C(1, 5)$

Translation: $(x, y) \rightarrow (x, y - 7)$

Reflection: in the line $x = 3$

SOLUTION
Begin by graphing $\triangle ABC$. Then, shift the triangle 7 units down to produce $\triangle A'B'C'$. Finally, reflect the triangle in the line $x = 3$ to produce $\triangle A''B''C''$.

Coordinates

<table>
<thead>
<tr>
<th>$\triangle ABC$</th>
<th>$\triangle A'B'C'$</th>
<th>$\triangle A''B''C''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(-5, 5)$</td>
<td>$A'(-5, -2)$</td>
<td>$A''(11, -2)$</td>
</tr>
<tr>
<td>$B(-3, 2)$</td>
<td>$B'(-3, -5)$</td>
<td>$B''(9, -5)$</td>
</tr>
<tr>
<td>$C(1, 5)$</td>
<td>$C'(1, -2)$</td>
<td>$C''(5, -2)$</td>
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</tbody>
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Exercises for Example 1

Sketch the image of $\triangle ABC$ after a glide reflection using the given transformations in the order they appear. Then, reverse the order of the transformations and sketch the image again. Determine if the order of the transformations affects the image.

1. $A(0, 0), B(0, 5), C(7, 0)$
   - Translation: $(x, y) \rightarrow (x + 3, y)$
   - Reflection: in the $x$-axis

2. $A(-3, 2), B(-1, -2), C(3, 2)$
   - Translation: $(x, y) \rightarrow (x - 4, y + 2)$
   - Reflection: in the line $x = 2$

3. $A(3, -1), B(7, -1), C(6, 2)$
   - Translation: $(x, y) \rightarrow (x - 1, y + 5)$
   - Reflection: in the line $y = -1$

4. $A(-4, 0), B(0, 7), C(3, 1)$
   - Translation: $(x, y) \rightarrow (x, y + 3)$
   - Reflection: in the line $x = 4$

Example 2

Finding the Image of a Composition

Sketch the image of $\overline{AB}$ after a composition of the given rotation and reflection.

$A(-5, 3), B(-3, 7)$

- Rotation: $90^\circ$ clockwise about the origin
- Reflection: in the line $y = 1$

Solution

Begin by graphing $\overline{AB}$. Then rotate the segment $90^\circ$ clockwise about the origin to produce $\overline{AB'}$. Finally, reflect the segment in the line $y = 1$ to produce $\overline{AB''}$.

Exercises for Example 2

Sketch the image of $\overline{AB}$ after a composition using the given transformations.

5. $A(-5, 5), B(-3, 2)$
   - Translation: $(x, y) \rightarrow (x + 8, y - 2)$
   - Reflection: in the $x$-axis

6. $A(0, -8), B(3, -4)$
   - Rotation: $180^\circ$ clockwise about the origin
   - Reflection: in the line $x = 3$

7. $A(6, -1), B(9, 4)$
   - Translation: $(x, y) \rightarrow (x - 8, y + 1)$
   - Reflection: in the $y$-axis

8. $A(3, 10), B(7, 5)$
   - Translation: $(x, y) \rightarrow (x - 4, y)$
   - Rotation: $90^\circ$ counterclockwise about the origin