GOAL
Identify and use translations in the plane.

VOCABULARY

A translation is a transformation that maps every two points $P$ and $Q$ in the plane to points $P'$ and $Q'$, so that the following properties are true:
1) $PP' = QQ'$ and 2) $PP' \parallel QQ'$, or $PP'$ and $QQ'$ are collinear.

A vector is a quantity that has both direction and magnitude, or size.

When a vector is drawn as ray $\overrightarrow{PQ}$, the initial point, or starting point, of the vector is point $P$ and the terminal point, or ending point, of the vector is point $Q$.

The component form of a vector combines the horizontal and vertical components.

EXAMPLE 1

Using Theorem 7.5

In the diagram, a reflection in line $k$ maps $\overline{AB}$ to $\overline{A'B'}$, a reflection in line $m$ maps $\overline{A'B'}$ to $\overline{A''B''}$, $k \parallel m$, $AW = 7$, and $ZA'' = 3$.

a. Name some congruent segments.


c. What is the length of $\overline{AA''}$?

SOLUTION

a. Here are some sets of congruent segments: $\overline{AB}$, $\overline{A'B'}$, and $\overline{A''B''}$; $\overline{BX}$ and $\overline{B'X}$; $\overline{BY}$ and $\overline{B''Y}$.

b. Yes, $WZ = XY$ because $WZ$ and $XY$ are opposite sides of a rectangle.

c. Because $AA'' = BB''$, the length of $\overline{AA''}$ is $7 + 7 + 3 + 3$, or 20 units.

Exercises for Example 1

In the diagram $k \parallel m$, $\triangle XYZ$ is reflected in line $k$, and $\triangle X'Y'Z'$ is reflected in line $m$.

1. Name two segments parallel to $\overline{YY''}$.

2. If the length of $\overline{ZZ''}$ is 6 cm, what is the distance between $k$ and $m$?

3. A translation maps $\triangle XYZ$ onto which triangle?

4. Which lines are perpendicular to $\overline{XX''}$?
Translations in a Coordinate Plane

Sketch a quadrilateral with vertices \(A(0, 4), B(-2, 1), C(0, -3),\) and \(D(3, 4)\). Then sketch the image of the quadrilateral after the translation \((x, y) \rightarrow (x + 2, y - 1)\).

**Solution**

Plot the points as shown. Shift each point 2 units to the right and 1 unit down to find the translated vertices.

**Exercises for Example 2**

In Exercises 5–8, copy figure \(PQRS\) and draw its image after the translation.

5. \((x, y) \rightarrow (x - 4, y + 1)\)
6. \((x, y) \rightarrow (x, y - 5)\)
7. \((x, y) \rightarrow (x - 2, y - 2)\)
8. \((x, y) \rightarrow (x + 7, y + 3)\)

Finding Vectors

In the diagram, \(\triangle ABC\) maps onto \(\triangle A'B'C'\) by a translation. Write the component form of the vector that can be used to describe the translation.

**Solution**

Choose any vertex and its image, say \(A\) and \(A'\). To move from \(A\) to \(A'\), you move 3 units to the right and 5 units down. The component form of the vector is \((3, -5)\).

**Exercises for Example 3**

In Exercises 9 and 10, write the component form of the vector that describes the translation which maps \(\triangle ABC\) onto \(\triangle A'B'C'\).

9. \(A(3, 6), B(1, 0), C(4, 8); A'(1, 2), B'(-1, -4), C'(2, 4)\)
10. \(A(-6, -2), B(-5, 3), C(1, -1); A'(-3, -5), B'(-2, 0), C'(4, -4)\)